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Covariant Action for the Super–Five–Brane of M–Theory

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Abstract

We propose a complete, d=6 covariant and kappa–symmetric, action for an M–theory five–brane propagating in $D = 11$ supergravity background.

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Among new types of super–p–branes [1]–[7], that have attracted a lot of attention during last several years, a five–brane [4, 5] of eleven–dimensional M-theory [3] is one of few for which the complete κ–invariant action has been unknown. In [8] a covariant action for the bosonic five–brane interacting with gravitational and antisymmetric fields of $D = 11$ supergravity was constructed, which completed partial results on the structure of the bosonic part of the action for the five–brane of M-theory [9]–[12].

In the present letter we propose a manifestly covariant κ–symmetric action for a five–brane propagating in $D = 11$ superspace of M–theory. Since the five–brane carries in its worldvolume a self–dual rank–two field, to construct the action we use a Lorentz–covariant approach to describing self–dual gauge fields proposed and developed in [13][1]. Only results are presented, while a detailed proof of κ–invariance is postponed to a forthcoming paper.

The five–brane action has the same form as in the bosonic case [8] but with $D = 11$ background fields replaced with superfields in curved $D = 11$ superspace parametrized by bosonic coordinates $X^{\underline{m}}$ ($\underline{m} = 0, 1..., 10$) and Grassmann spinor coordinates $\Theta^{\underline{\mu}}$ ($\underline{\mu}$=1,...,32) called altogether as $Z^{\underline{M}}$ [2]:

$$S = -\int d^6x \left[\sqrt{-det(g_{mn} + i\tilde{H}_{mn})} - \sqrt{-g}\frac{1}{4\partial_r a \partial^r a}\partial_l a(x) H^{*lmn} H_{mnp} \partial^p a(x)\right] - \int \left[C^{(6)} + \frac{1}{2}F \wedge C^{(3)}\right]. \tag{1}$$

In (1) small x^m (m=0,1,...,5) parametrize five–brane worldvolume; $a(x)$ is an auxiliary scalar field which ensures $d = 6$ general coordinate invariance of the action [8];

$$g_{mn}(x) = E^{\underline{a}}_m(x)\eta_{\underline{ab}}E^{\underline{a}}_n(x) \qquad (\underline{a}, \underline{b} = 0, ..., 10) \tag{2}$$

is an induced worldvolume metric constructed of components of the $D = 11$ supervielbeins $E^{\underline{A}} = dZ^{\underline{M}}E^{\underline{A}}_{\underline{M}}$ pulled back to the worldvolume ($\underline{A} = (\underline{a}, \underline{\alpha})$ denote tangent superspace indices.). In the flat target superspace the metric takes the form

$$g_{mn}(x) = \partial_m \Pi^{\underline{m}}(x) g_{\underline{mn}} \partial_n \Pi^{\underline{n}}(x) \qquad (\Pi^{\underline{m}} = dX^{\underline{m}}(x) - id\Theta\Gamma^{\underline{m}}\Theta). \tag{3}$$

$A_{mn}(x)$ is a worldvolume self–dual (or so–called chiral) field with the field strength $F_{mnl} = 2(\partial_l A_{mn} + \partial_m A_{nl} + \partial_n A_{lm})$, (note that a generalized self–duality condition for A_{mn} arises from (1) as an equation of motion [12, 8]);

$$H_{lmn}(x) = F_{lmn} - C^{(3)}_{lmn}, \qquad \tilde{H}_{mn} \equiv \frac{1}{\sqrt{-(\partial a)^2}} H^*_{mnl}\partial^l a(x), \qquad H^{*mnl} = \frac{1}{3!\sqrt{-g}}\varepsilon^{mnlpqr}H_{pqr}, \tag{4}$$

and $C^{(3)}_{lmn}$ and $C^{(6)}_{lmnpqr}$ are pullbacks into worldvolume of superforms $C^{(3)}(X, \Theta)$ and $C^{(6)}(X, \Theta)$ of $D = 11$ supergravity whose field strengths are dual to each other in the following sense [14]

$$^*dC^{(3)} = dC^{(6)} + \frac{1}{2}C^{(3)}R^{(4)} \equiv R^{(7)}, \qquad R^{(4)} \equiv dC^{(3)}, \tag{5}$$

[1]When this paper was prepared for publication we learned that in a noncovariant formulation [12] the proof of the κ-invariance of a super–five–brane action was also carried out by J. H. Schwarz with collaborators [15].

[2]We use underlined indices for denoting the coordinates of target superspace and not underlined ones for the coordinates of the five–brane worldvolume. The signature of the metrics is chosen almost negative, the external derivative acts from the right and the $D = 11$ gamma–matrices are imaginary

(where * denotes eleven–dimensional "bosonic" Hodge operation accompanied by $(\Gamma_{\underline{ab}})_{\underline{\alpha\beta}} \quad \rightarrow \quad (\Gamma_{\underline{a_1}\ldots\underline{a_5}})_{\underline{\alpha\beta}}$). The $D = 11$ supergravity background fields are assumed to satisfy the constraints:

$$T^{\underline{a}} = \mathcal{D}E^{\underline{a}} = -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}}\Gamma^{\underline{a}}_{\underline{\alpha\beta}} + E^{\underline{b}} \wedge E^{\underline{\beta}}T^{\underline{a}}_{\underline{b\beta}} + \frac{1}{2}E^{\underline{b}} \wedge E^{\underline{c}}T^{\underline{a}}_{\underline{bc}},$$

$$R^{(4)} = dC^{(3)} = \frac{1}{2}E^{\underline{b}} \wedge E^{\underline{a}} \wedge E^{\underline{\alpha}} \wedge E^{\underline{\beta}}(\Gamma_{\underline{ab}})_{\underline{\alpha\beta}}$$

$$+ \frac{1}{4!}E^{\underline{a}} \wedge E^{\underline{b}} \wedge E^{\underline{c}} \wedge E^{\underline{d}}R_{\underline{dcba}}, \qquad (6)$$

$$R^{(7)} = \frac{i}{5!}E^{\underline{a_1}} \wedge \ldots \wedge E^{\underline{a_5}} \wedge E^{\underline{\alpha}} \wedge E^{\underline{\beta}}(\Gamma_{\underline{a_1}\ldots\underline{a_5}})_{\underline{\alpha\beta}} + \mathcal{O}\left((E^{\underline{a}})^6\right).$$

Local transformations which leave the action (1) invariant were discussed in [8], so we only present a local symmetry which reflects an auxiliary role of $a(x)$

$$\delta a(x) = \varphi(x),$$

$$\delta A_{mn} = \frac{\varphi(x)}{2(\partial a)^2}(H_{mnp}\partial^p a - \mathcal{V}_{mn}), \qquad (7)$$

where

$$\mathcal{V}^{mn} \equiv -2\sqrt{\frac{(\partial a)^2}{g}}\frac{\delta\sqrt{-\det(g_{pq} + i\tilde{H}_{pq})}}{\delta \tilde{H}_{mn}}.$$

The second integral in (1) is the Wess–Zumino term $\int \mathcal{L}^{(6)}_{WZ}$. Its external derivative (required for proving the κ–invariance) is a closed 7-superform

$$d\mathcal{L}^{(6)}_{WZ} = R^{(7)} + \frac{1}{2}H \wedge R^{(4)}, \qquad (8)$$

which in flat $D = 11$ superspace takes the following form:

$$d\mathcal{L}^{(6)}_{WZ} = \frac{i}{5!}\Pi^{\underline{m}_1} \wedge \ldots \wedge \Pi^{\underline{m}_5}d\Theta\Gamma_{\underline{m}_1\ldots\underline{m}_5}d\Theta + \frac{1}{2}H \wedge \Pi^{\underline{m}_1} \wedge \Pi^{\underline{m}_2}d\Theta\Gamma_{\underline{m}_1\underline{m}_2}d\Theta. \qquad (9)$$

Note that the coefficient in front of the Wess–Zumino term is fixed already in the purely bosonic case by the requirement of the invariance of (1) under (7) (see [8]).

As in the case of the D-branes [6] an indication that the action (1) is invariant under fermionic κ–transformations is the existence of a matrix $\bar{\Gamma}$ whose square is the unit matrix. In our case a relevant matrix has the following form:

$$\sqrt{-\det(g + i\tilde{H})}\bar{\Gamma} = \sqrt{-g}[\Gamma^{(6)} + \frac{i}{2\sqrt{-(\partial a)^2}}\tilde{H}^{mn}\Gamma_{mn}\Gamma_p\partial^p a$$

$$+ \frac{1}{8(\partial a)^2}\partial^{m_1}a\varepsilon_{m_1\ldots m_6}\tilde{H}^{m_2m_3}\tilde{H}^{m_4m_5}\Gamma^{m_6}\Gamma_p\partial^p a], \qquad (10)$$

where

$$\Gamma_m = \Gamma_{\underline{a}}E^{\underline{a}}_m \qquad (\Gamma_m = \Gamma_{\underline{n}}\Pi^{\underline{n}}_m \text{ in flat target superspace})$$

are the pullbacks into the worldvolume of the $D = 11$ gamma–matrices, $\Gamma^{(n)}$ is the antisymmetriezed product of n Γ_m.

The action (1) indeed possesses κ–invariance, the κ–transformations of the worldvolume fields being:

$$i_\kappa E^{\underline{\alpha}} = \delta_\kappa Z^M E_M^{\underline{\alpha}} = \kappa^{\underline{\beta}}(1+\bar{\Gamma})_{\underline{\alpha}}^{\ \underline{\beta}}, \qquad i_\kappa E^{\underline{a}} = 0, \qquad \delta_\kappa g_{mn} = -4iE_{\{m}\Gamma_{n\}}i_\kappa E,$$
$$\delta_\kappa H = -i_\kappa dC^{(3)}, \tag{11}$$

or in the flat case

$$\delta_\kappa \Theta^{\underline{\mu}} = \kappa^{\underline{\nu}}(1+\bar{\Gamma})_{\underline{\nu}}^{\ \underline{\mu}}, \qquad \delta_\kappa \Pi^{\underline{m}} = -2id\Theta\Gamma^{\underline{m}}\delta_\kappa\Theta, \qquad \delta_\kappa g_{mn} = -4i\partial_{\{m}\Theta\Gamma_{n\}}\delta_\kappa\Theta,$$
$$\delta_\kappa H = -\Pi^{\underline{n}} \wedge \Pi^{\underline{m}} \wedge d\Theta\Gamma_{\underline{mn}}\delta_\kappa\Theta. \tag{12}$$

Because of a Born–Infeld–like form of (1) the check of the κ–invariance of the five–brane action is carried out using the way analogous to that for the Dirichlet branes [6]. A difference is in the presence in the first integral of (1) of the term quadratic in H whose κ–variation contributes to the variation of the Wess–Zumino term. As in the bosonic case [8], upon a double dimensional reduction the D=11 super–five–brane should reduce to a dual version of a D=10 Dirichlet super–4–brane. A detailed analysis of the action (1) will be made in a forthcoming paper.

In conclusion we have constructed the covariant action for the five–brane of M–theory which is invariant under the κ–symmetry transformations and contains the auxiliary scalar field $a(x)$. The role of the auxiliary field is to ensure the covariance of the model under $d = 6$ worldvolume diffeomorphisms (which makes the analysis of the model much simpler) [3], its variation does not lead to independent field equations, and $\partial_m a(x)$ cannot square to zero [13, 8]. Thus the presence of this field in the action might be a manifestation of nontrivial topological features of the five–brane and M–theory itself.

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